The overall performance of an instrument is based on its static and dynamic characteristics. It indicates how well the instrument measures the desired input and rejects the spurious (or undesired) inputs.

If the sensor is made to measure constant or slowly varying quantities, its performance can be evaluated with only the static characteristics.

The dynamic description is necessary if we need to measure rapidly varying quantities. This involves the study of the instrument input-output relations and the use of differential equations.

### 3.1 Static Calibration

**Static calibration** refers to the input-output relations obtained when only one input of the instrument is varied at a time, all other inputs being kept constant.

To each constant input, which is measured with another instrument, we associate the sensor output. The plot of these points is the sensor static characteristic for that specific input.
Steps for calibration:

1. Identify all the possible inputs of the instrument.
2. Decide which of the inputs will be significant in your application.
3. Determine the apparatus and methods to control (vary or maintain constant) all significant inputs over the desired range.
4. By varying one input and holding the other inputs constant, develop the sensor input-output relations.

Example: Calibration of the pressure gage sensor

The objective in this example is to determine the relationship between the desired input (pressure) and the output (scale reading).

The first step of the calibration process requires identifying the desired, interfering and modifying inputs of the pressure gage.

In the second step you must determine how, or in what conditions, you are going to use the sensor.

For example, what will be the surrounding temperature? If the temperature (which is an interfering input for this sensor) varies over a large range during the normal use of the sensor, maybe you will have to do the (desired-input / output) calibration for different value of the temperature.

Then you must ensure that, by choosing the appropriate experimental conditions, all the inputs of the pressure gage, except the fluid pressure, are kept constant.
The fluid pressure (true value) must be varied with another instrument, in increments, over some range, causing the measured value also to vary:

The average calibration curve is taken as a straight line of equation:

\[ q_o = mq_i + b \]

\( m \) and \( b \) can be obtained with the least-squares criterion. In this example, \( m = 1.08 \) and \( b = -0.85 \) kPa
The reading $q_o$ allow us to write an estimate of the true pressure as

$$q_i = \frac{(q_o + 0.85)}{1.08}$$

However, this value, obtained from the least-squares line, must have some plus-or-minus error, given by the standard deviation $s_{qi}$,

$$s_{qi}^2 = \frac{1}{N} \sum [(q_o - b)/m - q_i]^2$$

$$s_{qi} = 0.18 \text{ kPa}$$

Thus if the reading from the gage is 4.32 kPa, our estimate of the true pressure would be

$$q_i \pm 3s_{qi} = \frac{(4.32 + 0.85)}{1.08} \pm (3 \times 0.18)$$

$$= 4.79 \pm 0.54 \text{ kPa} \quad \text{(with a $\pm 3s_{qi}$ limits)}$$

Note: By taking an error of three times the standard deviation, we ensure that the true pressure will be in the defined range $[4.79 - 0.54 ; 4.79 + 0.54]$ with a probability of 99.7%.

- The total error of measurement is in two parts:
  - the bias, 0.47 kPa \hspace{1cm} (= 4.79 - 4.32)
  - the imprecision, $\pm 0.54$ kPa

These two terms are discussed in the next paragraph.
3.2 Accuracy, Precision, and Bias

The **accuracy** (lack of error) of an instrument can be evaluated in terms of the concepts of precision and bias.

In practice, every device will produce an error on the measured quantities. Knowing the accuracy of that device will allow us to put bounds on the error.

The **precision** of a measuring instrument is the degree to which it produces similar results for the same inputs on a number of occasions.

The **bias** on a measurement is the part of the error that can be removed by calibration. It has the same value for all measurements.

The **imprecision** (or *random error*) is a type of error that is not known precisely for a given measurement, just bounded. It is different for every reading and we cannot remove it.

The total **inaccuracy** of the measuring instrument is defined by the combination of bias and imprecision.
### 3.3 Static Sensitivity

The **sensitivity** of an instrument refers to its ability to detect changes in the measured quantity.

It can be defined as the slope of the calibration curve if the input/output relationship is linear.

The output quantity must be taken as the actual physical output:

Which is an angular displacement for the pressure gage: 5 degrees/kPa, if the spacing between two marks is 5 degrees.

Thus the static sensitivity of the pressure gage is $5 \times 1.08$ (slope of the calibration curve) = $5.4^\circ$/kPa

The sensitivity of an instrument refers to the true quantity that is being measured ($q_i$ in our example).

The sensitivity of the instrument to interfering and/or modifying inputs can also be of interest.
3.4 Threshold, Resolution, and Hysteresis

**Hysteresis** causes a difference in the output curve of a sensor when the direction of the input has been reversed. Common causes of hysteresis are mechanical strain and friction.

![Figure 3.2 Hysteresis](image)

The **threshold** is the minimum value of the input below which no output is detected. It defines the smallest measurable input, starting from rest.

The **resolution** is the minimum increment of input (starting from nonzero) below which no output is detected (zoom on the figure). It defines the smallest measurable change of the input.

![Figure 3.3 Threshold and Resolution](image)